Control Engineering Practice 27 (2014) 54–60

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

# Model predictive control based on an integrator resonance model applied to an open water channel

Peter-Jules van Overloop [a,*], Klaudia Horváth [b], Boran Ekin Aydin [a]

[a] Delft University of Technology, Section of Water Resources Management, Stevinweg 1, 2628 CN, Delft, The Netherlands
[b] Technical University of Catalonia, Department of Hydraulic, Maritime and Environmental Engineering, C. Jordi Girona 1, D1/208, Barcelona 08034, Spain

ARTICLE INFO

*Article history:*
Received 1 February 2013
Accepted 2 March 2014
Available online 26 March 2014

*Keywords:*
Model predictive control
Open water channel
Canal model
Simplified model
Resonance

ABSTRACT

This paper describes a new simplified model for controller design of open water channels that are relatively short, flat and deep: the integrator resonance model (IR model). The model contains an integrator and the first resonance mode of a long reflecting wave. The paper compares the integrator resonance model to the simplified models: integrator delay, integrator delay zero and filtered integrator delay and to the high-order linearized Saint-Venant equations model. Results of using the integrator resonance model in a model predictive controller applied in closed loop on a high-order non-linear Saint-Venant model of the first pool of the laboratory canal at Technical University of Catalonia, Barcelona are compared to the results of using the other simplified models in MPC. This comparison shows that the IR model has less model mismatch with the high order model regarding the relevant dynamics of these typical channels compared to the other simplified models. It is demonstrated that not considering the resonance behavior in the controller design may result in poor performance of the closed loop behavior. In order to demonstrate the validity of the simulation model used in this study, the controller using the IR model is also tested on the actual open water channel and compared to the results of the high-order non-linear Saint-Venant simulation model. The results of this comparison show a close resemblance between simulation model and real world system.



## 1. Introduction

In order to increase water delivery efficiency, throughout the world, control of open water channels such as irrigation canals is implemented, often referred to as canal automation. A common control configuration is distant downstream control where the water level $h_2$ downstream in the open water channel needs to be kept as close as possible to a setpoint by adjusting the flow $Q_1$ of the hydraulic structure upstream in the channel (see Fig. 1). For open water channels that are short, flat and deep this can be complicated due to the occurrence of resonances (Schuurmans, 1997). These resonances are badly damped long waves that reflect on the ends of the open water channel. Channels that are short, flat and deep have a small integrated friction force over the length of the pool, which means it is easy for waves to travel up and down the pool a number of times before settling. The characterization in short versus long, flat versus steep and deep versus shallow is hard to define in standard rules, due to the complexity of the dynamic behavior of open water channels. van Overloop (2006) attempts to capture the sensitivity for resonance waves as a function of length, width, friction coefficient, flow and average depth of an open water channel. Litrico and Fromion (2009) prove that resonance waves are also present in long, steep and shallow open water channels, but that they do not show up in measurements as they are damped significantly.

The first resonance mode, as depicted in Fig. 1, is troublesome for distant downstream control as its peak is in 180° phase lag with the control input. The gain margin criterion for designing feedback controllers dictates the controlled system gain at that frequency to be smaller than 0.5, in order to be robust against instability. Another explanation why it is hard to deal with this first resonance mode is that it does not make sense to decrease the flow $Q_1$, when the water level $h_2$ is higher than the setpoint, because in fact the flow at the upstream side of the pool is already lower due to the oscillation and needs not be lowered more.

In order to avoid unstable closed loop control, there are three ways to deal with the resonance in the controller design. First, the resonance can be accepted and, consequently, obeying the gain margin criterion will result in a low performing closed loop behavior. Second, the resonance can be filtered before it enters the controller, allowing for a higher performance. In this case, the resonance is present, but the controller does not react to it in order

* Corresponding author. Tel.: +31 15 2782345; fax: +31 15 2785559.
*E-mail addresses:* p.j.vanoverloop@citg.tudelft.nl (P.-J. van Overloop), klaudia.horvath@upc.edu (K. Horváth).

http://dx.doi.org/10.1016/j.conengprac.2014.03.001
0967-0661/

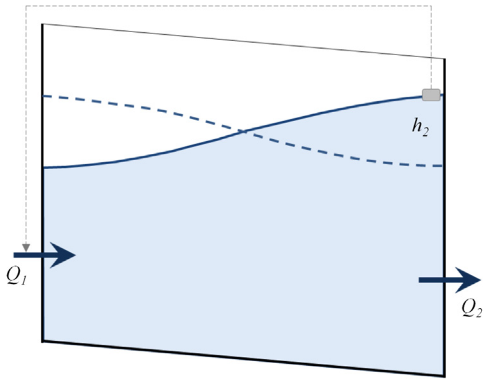


**Fig. 1.** Resonance-sensitive open water channel including distant downstream control loop.

to avoid instability. Third and the last, the resonance can be included in the controller model, which means the controller avoids triggering the resonance mode as much as possible. In this paper, these different ways of controller design are analyzed and evaluated on an actual open water channel that is very sensitive to resonance waves. The innovative aspect of this research is that it proposes a new simplified model that models the first resonance mode as part of the controller model: the integrator resonance model (IR model).

## 2. Simplified models of open water channels

In order to develop controllers for open water channels, simplified models need to be developed. Many standard controller design algorithms for feedback and feedforward control require low order linear models (Vandevegte, 1990). Low order and linear models are also important for real-time optimizing controllers, such as model predictive control, in order to achieve tractability and convexity (Camacho & Bordons, 2004).

Over the past two decades, in the field of control of open water channels, various types of simplified models have been developed. Schuurmans (1997) proposes the integrator delay (ID) model, which consists of a delay part describing the upstream uniform flow part and an integrator describing the downstream storage volume of which the water level needs to be kept at setpoint. This model describes the low frequency behavior accurately, but does not contain resonance modes. For long, steep and shallow open water channels, this model performs excellent. By simplifying the linearized Saint-Venant equations, Litrico and Fromion (2004) arrive at the conclusion of adding a zero to the ID model in order to model the high frequency behavior. This integrator delay zero (IDZ) model captures the average behavior of the resonances, but unfortunately not the peaks of which the first one is so important for stable controller design of distant downstream control. Weyer (2001) attempts to capture the first resonance by proposing a third-order model with delay and fitting this model to measured data using system identification algorithms. In van Overloop et al. (2010) it is demonstrated that this procedure may result in the underestimation of the first resonance peak due to the influence of the second and even higher resonance modes that are present in the measurements and may not be completely filtered out. The system identification algorithm tries to fit the resonator to be optimal for both the first peak and the higher harmonics that are remaining in the signals, reducing the value of the identified peak of the important first resonance mode. A simplified model for open water channels that focuses on the always present integrator and only the first resonance mode has not been assessed in the literature before.

## 3. Open water channel dynamics of resonance-sensitive open water channels

The Saint-Venant equations, calibrated on the friction parameter, describe the dynamic behavior of water flow in an open water channel accurately (Chow, 1959). A discretized and linearized Saint-Venant model of a short, flat and deep pool presented in the frequency domain demonstrates clearly the integrator at low frequencies and the resonance peaks at higher frequencies (see solid line in Fig. 2). In van Overloop et al. (2010) the Laplace transfer function from an inflow $Q_1$ to the downstream water $h_2$ is derived. This transfer function is a third-order model without time delay consisting of an integrator with a gain of the reciprocal of the storage area $A_s$ and a damped oscillator characterized by the natural frequency $\omega_0$ and magnitude peak $M$ of the first resonance

$$H_{IR}(s) = \overbrace{\frac{1}{A_s \cdot s}}^{\text{Integrator}} \cdot \overbrace{\frac{{\omega_0}^2}{s^2 + \frac{s}{A_s \cdot M} + {\omega_0}^2}}^{\text{Resonance}} \tag{1}$$

The parameters' storage area $A_s$, the natural frequency $\omega_0$ and magnitude peak $M$ of the first resonance of this model structure can be estimated using different procedures, e.g. from a step response, system identification using a chirp signal or a random binary signal around an initial estimate of the natural frequency. An important notice is that this model is a linearization of the non-linear Saint-Venant equations, so the parameters are different in different working points (see for example Table 1). The most important variables that determine the working points are the

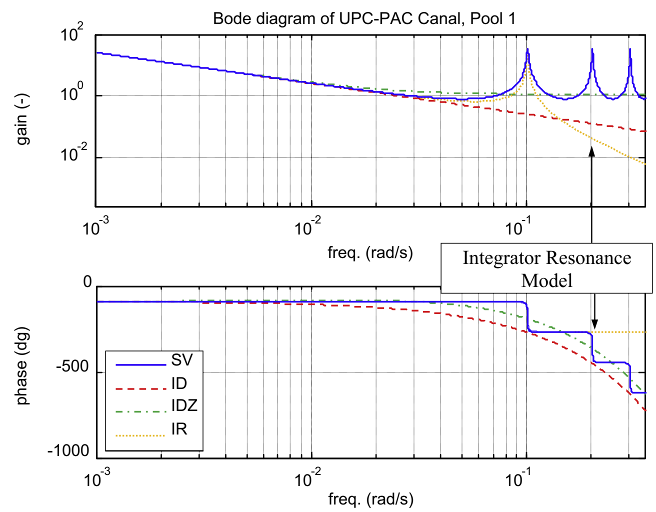


**Fig. 2.** Bode diagram of high-order linearized Saint-Venant model (solid line), integrator delay model (dashed line), integrator delay zero model (dash dotted line) and integrator resonance model (dotted line) modeling the laboratory open water channel.

**Table 1**
Properties of the first resonance of the first pool of laboratory canal UPC-PAC.

| Flow (l/s) | Frequency $\omega_0$ (rad/s) | Magnitude $M$ |
|---|---|---|
| 10 | 0.1011 | 35.09 |
| 30 | 0.1011 | 11.77 |
| 50 | 0.1010 | 7.11 |
| 70 | 0.1008 | 5.14 |
| 90 | 0.1006 | 4.05 |
| 110 | 0.1003 | 3.37 |
| 130 | 0.0999 | 2.91 |
| 150 | 0.0997 | 2.59 |

water level at the downstream side of the open water channel and the flow through the open water channel. As the goal of a controller is to keep the downstream water level at setpoint and well-designed controllers are capable of doing so, the main variable that defines the working point is the flow.

In order to see the difference in dynamic behavior, various models of a resonance-sensitive open water channel are compared in the frequency domain. As example, the first pool of the laboratory canal assessed in this paper is taken at a flow $Q_1$ of 10 l/s and a depth $D$ of 0.80 m. The channel is fully under backwater, which means waves can reflect easily forth and back. As a reference, the discretized linearized Saint-Venant model is taken and compared to the integrator delay model, integrator delay zero model and the integrator resonance model. The integrator delay model uses the total surface area of the open water channel as a storage area and the delay time $\tau_d$ derived from the celerity $\tau_d = L/\sqrt{g \cdot D}$ with $L$ is the length of the pool and $g$ is the gravitational acceleration. The integrator delay zero model is an analytical solution (Litrico & Fromion, 2004) using the same dimensions of the channel as used in the Saint-Venant model. The integrator resonance model uses again the total surface area as the storage area and uses the frequency and the magnitude of the first resonance peak of the Saint-Venant model to fit the IR model. This frequency is $\omega_0 = 0.101$ rad/s and the magnitude of the first resonance peak is $M(\omega_0) = 35.09$ for this flow condition (see also Table 1). In Fig. 2, the Saint-Venant model together with the ID, IDZ and IR model are shown.

Fig. 2 shows that the Saint-Venant model (solid line) has a $-90^\circ$ phase shift originating from the integrator between flow as an input and water level as an output variable and an extra $-180^\circ$ step per extra resonance mode. The integrator delay (dashed line) and integrator delay zero (dash dotted line) models try to mimic the resonance phase shift by adding an extra time delay. No resonance peaks can be seen in the gain graph for these two models. The integrator resonance model (dotted line) is accurate in the low frequencies passed the first resonance. This is the part of the spectrum that is important given the stability issues that may arise for resonance-sensitive pools. Hence, the IR model captures the low-frequency behavior and the first resonance peak, while ID and IDZ do not model this important peak.

## 4. The laboratory canal

The laboratory canal UPC-PAC (Technical University of Catalonia – Control Algorithms Test Canal) is located in Barcelona, at the Northern Campus of the University. The facility occupies an area of 22.5 m × 5.4 m, being 220 m long and having serpentine shape with a rectangular cross section. It is 1 m deep, 0.44 m wide, and has a zero bottom slope. It contains three motorized vertical sluice gates, nine water levels sensors, and four rectangular weirs. With the help of these structures it is possible to use different configurations from one pool to three pools. The maximum discharge is 150 l/s. In this paper, the first pool of this canal is modeled and controlled; its length is 87 m. At the upstream end there is an undershot gate that separates the pool from a constant level reservoir and at the downstream end there is an undershot gate as well. Sepúlveda Toepfer (2008) gives a concise description of the laboratory canal.

In order to see the dependency of the first resonance mode given the different flows that may occur in the canal pool, the frequency and magnitude of this resonance are estimated and presented in Table 1 using the high-order linearized Saint-Venant model of the pool.

The frequency of the resonance wave is only changing slightly, while the peak magnitude is decreasing with increasing flow and is very high at the lowest flow of 10 l/s. As the resonance is most prominent at this low flow, the tests are executed in this working point.

## 5. Controller design

Model predictive control is a control methodology that has proven to be very suitable for the control of open water channels and is also applied in this research. Figueiredo, Botto, and Rijo (2013), Li and De Schutter (2011), and Pascual et al. (2013) are all examples of the application of model predictive control on water systems that, unlike the open water channel assessed in this research, are not dominated by strong resonance effects.

For the first pool of the laboratory canal, four model predictive controllers are designed according to van Overloop (2006) based on four different internal models. This controller is a finite horizon, linear time-invariant MPC configured as a quadratic programming problem. The goal of the research is to demonstrate that a mismatch between internal model and actual system at important frequencies can lead to poor performance or even instability in closed loop control. Constraints are left out of the optimization problem, as they do not add much relevance to the tests. The downstream flow $Q_2$ is the disturbance flow, while the upstream flow $Q_1$ is controlled by MPC. The models used are as follows:

- Integrator delay (MPC-ID) based on Schuurmans (1997):

$$h_2 = \frac{e^{-\tau_d s}}{A_s \cdot s} \cdot Q_1 - \frac{1}{A_s \cdot s} \cdot Q_2 \tag{2}$$

  where $A_s = 38.28$ m$^2$ and $\tau_d = 31.00$ s.
- Integrator delay zero (MPC-IDZ) based on Litrico and Fromion (2004):

$$h_2 = \frac{1 + z_1 s}{A_s s} e^{-\tau_d s} Q_1 - \frac{1 + z_2 s}{A_s s} Q_2, \tag{3}$$

  where $A_s = 38.28$ m$^2$, $\tau_d = 30.74$ s, $z_1 = 43.41$ and $z_2 = 31.06$.
- Integrator delay in series with a low-pass Filter (MPC-IDF) based on Schuurmans (1997):

$$h_2 = \frac{e^{(-\tau_d - \tau_f)s}}{A_s \cdot s} \cdot Q_1 - \frac{e^{-\tau_f s}}{A_s \cdot s} \cdot Q_2 \tag{4}$$

  where $A_s = 38.28$ m$^2$, $\tau_d = 31.00$ s and $\tau_f = 70.00$ s is added as an extra delay time in order to compensate for the extra delay time caused by the filter $F = 1/T_f \cdot s + 1$where $T_f$ is 120.4 s.
- Integrator resonance (MPC-IR):

$$h_2 = \frac{\omega_0{}^2}{A_s s^3 + \frac{s^2}{M} + A_s \cdot \omega_0{}^2 \cdot s} \cdot Q_1 - \frac{2 \cdot s^2 + \frac{2}{A_s \cdot M} \cdot s + \omega_0{}^2}{A_s s^3 + \frac{s^2}{M} + A_s \cdot \omega_0{}^2 \cdot s} \cdot Q_2 \tag{5}$$

  where $A_s = 38.28$ m$^2$, $\omega_0 = 0.101$ rad/s and $M = 35.09$.

The models are setup in state-space form which enables a straightforward implementation of the models in the MPC over the prediction horizon.

The control objective is to keep the downstream water level $h_2$ constant, while disturbance flow steps occur on $Q_2$. In all cases, a 10 s sampling time was used and a prediction horizon of 20 steps (3.3 min). The objective function over the prediction horizon

$$J = \sum_{i=1}^{20} \{W_e \cdot e(k+i)^2 + W_{e_{sum}} \cdot e_{sum}(k+i)^2 + W_{\Delta Q} \cdot \Delta Q_1 (k-1+i)^2\} \tag{6}$$

$$e(k) = h_2(k) - h_{2,ref} \tag{7}$$

$$e_{sum}(k) = e_{sum}(k-1) + e(k) \tag{8}$$

is minimized on the change of inflow $\Delta Q_1$. The penalty $W_e$ on the water level deviation $e$ from the setpoint $h_{2,ref}$=0.80 m is 100, the penalty $W_{esum}$ on the sum of the water level deviation $e_{sum}$ is 0.04, while the penalty $W_{\Delta Q}$ on the upstream flow change $\Delta Q_1$ was chosen different for each model in order to get the highest performance. The values for the penalties on the water level deviations and the sum of the water level deviations were selected based on trial and error. The penalties on the flow change were selected for each model according to the following trial-and-error procedure:

1. Select one of the four simplified models.
2. Start with a very high penalty on the flow change $W_{\Delta Q}$ of 250,000. Run the closed loop simulation of the high-order Saint-Venant model with MPC using the selected simplified model (the results for all models showed a very slow response with this high penalty).
3. Decrease the penalty stepwise in order to increase the performance.
4. Rerun the close loop simulation with the new penalty.
5. Repeat steps 3 and 4 until visually high frequent oscillations in the control flow can be distinguished. Use as final values for the model selected the penalties of the previous run in which these undesired oscillations did not show up yet.

The final value for $W_{\Delta Q}$ is 111,111 for ID, 250,000 for IDZ, 3460 for IDF and 10,000 for IR. For the IDF model it is possible to arrive at the smallest penalty due to the fact that the oscillations are filtered. The second smallest penalty is achieved by the IR model since it includes the first oscillation mode and the controller is internally able to counteract on this.

## 6. Simulation results

The simulation results presented are the unsteady solution of the full non-linear Saint-Venant equation using the SIC (simulation of irrigation canals) software package (Malaterre & Baume, 1998). The following experiment is carried out with all the models: at 30 min the discharge changes from 10 l/s to 20 l/s as a known disturbance. At 120 min, the discharge changes to the original value of 10 l/s, but now this is unknown by the controller.

### 6.1. ID model

The water level deviations caused by the known disturbance step are clearly smaller than the ones of the unknown disturbance. Oscillations in the water levels due to the resonances are visible but these are not harmful. The control flow does not react on these oscillations (that are not part of the ID model) only because the control has a high penalty on the flow changes. A lower penalty on the flow change directly leads to undesired oscillations in the control flow that will cause wear and tear in the long run (Fig. 3).

### 6.2. IDF model

Again, the water level deviations caused by the known disturbance step are smaller than the ones of the unknown disturbance. Oscillations in the water levels are first filtered out before the water level signal enters the controller. In this way, the controller does not react on the oscillations and a much lower penalty on the flow change can be used. Despite this much lower penalty, the performance is not higher compared to using the ID model without the filter (Fig. 4).

### 6.3. IDZ model

Also here, the water level deviations caused by the known disturbance step are smaller than the ones of the unknown disturbance. The performance is comparable to the performance when using the ID model, although the control flow shows a bit more overshoot and has less damping (Fig. 5).

### 6.4. IR model

The water level deviations when using the IR model of the known and unknown disturbance steps are much smaller than when using the other models. This is due to the small penalty on the flow change that can be used for this model. This small penalty can only be used because the internal model fits the real system well. Using the same small penalty for the ID and IDZ model resulted in an unstable closed loop system.

For all models used, the high frequent oscillations in the water levels are caused by the resonances in the system itself and not by the controllers. These oscillations are unavoidable and harmless,

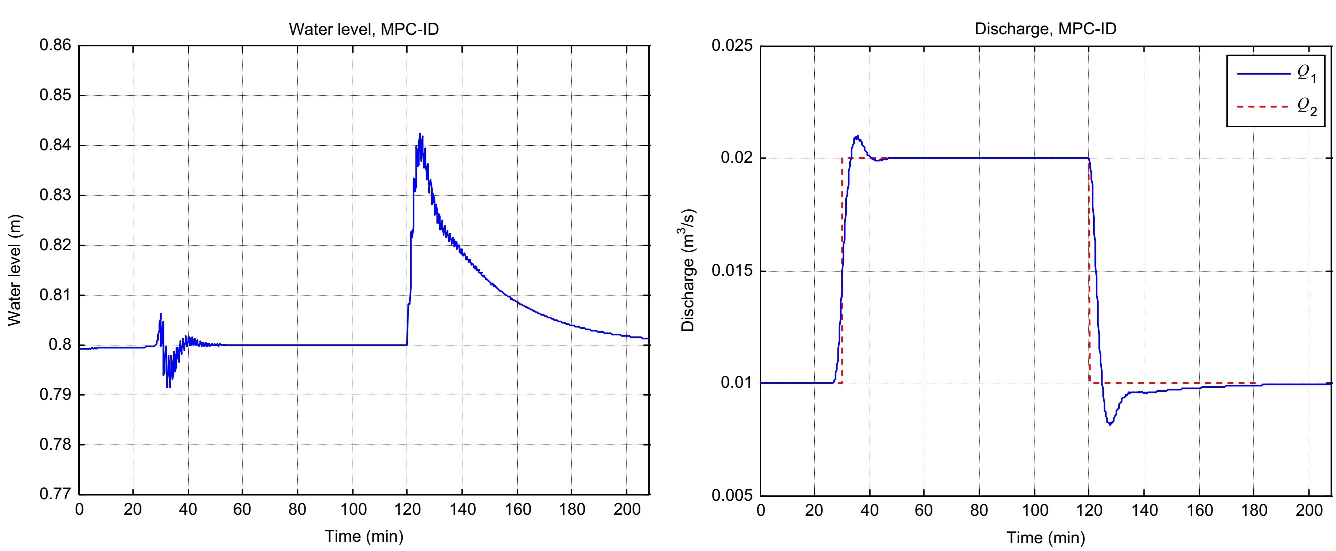


**Fig. 3.** Simulated distant downstream water level response ($h_2$) and control flow ($Q_1$) reacting on known and unknown disturbance of the model predictive controller using the integrator delay model.

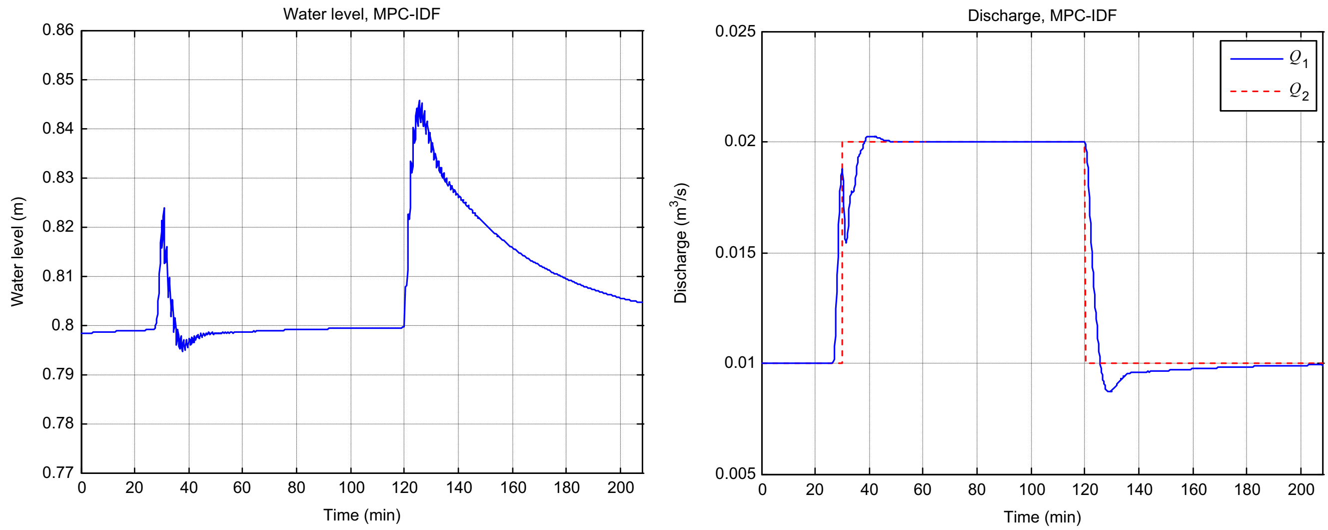


**Fig. 4.** Simulated distant downstream water level response ($h_2$) and control flow ($Q_1$) reacting on known and unknown disturbance of the model predictive controller using the integrator delay model with filter.

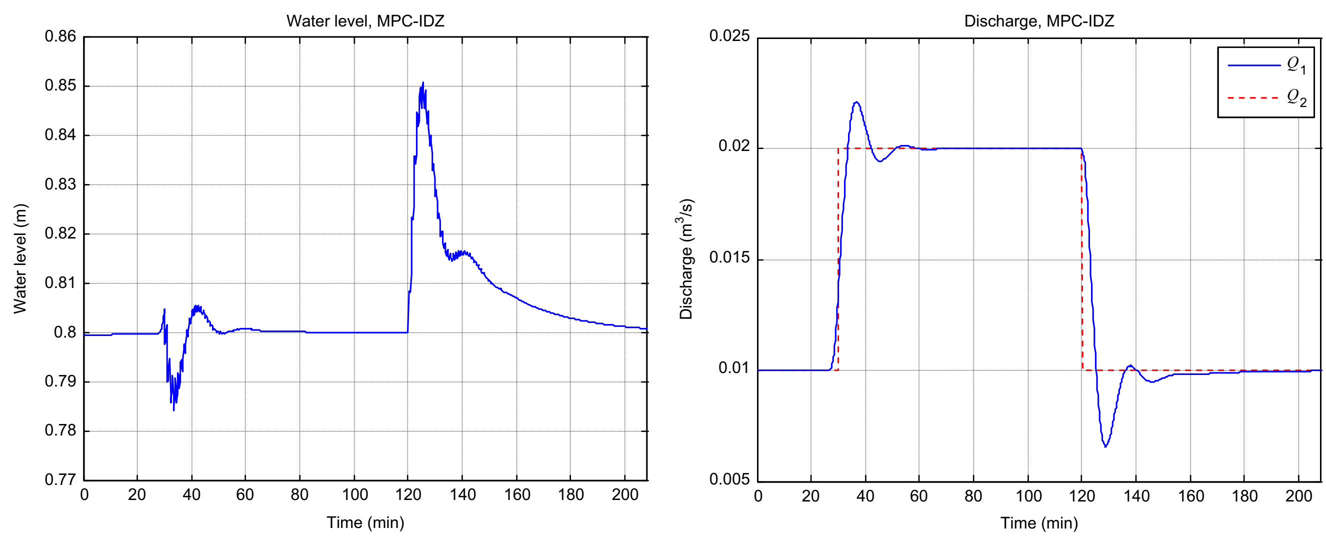


**Fig. 5.** Simulated distant downstream water level response ($h_2$) and control flow ($Q_1$) reacting on known and unknown disturbance of the model predictive controller using the integrator delay zero model.

while if these oscillations would show up in the control flow, they would be unacceptable for reasons of wear and tear. As expected, all results show a small water level error when the disturbance is known by the controller and a larger deviation for the unknown case, where the controller needs to depend on its feedback functionality (Fig. 6).

## 7. Results on actual open water channel

As the IR model is the focus of this research, this model is also tested on the actual laboratory canal in order to check if it also behaves properly in a real world application. The same experiment as described for the IR model above is conducted with the difference that the changes happen at 10 min and 40 min.

Figs. 7 and 8 indicate that the model predictive controller based on the integrator resonance model clearly resembles the model results. The differences are due to the constraints in the hardware being the minimum gate movement, and the water level and gate position measurement errors. These hardware constraints cause the water levels to fluctuate around the setpoint.

## 8. Discussion

As can be seen from the results, it is possible to design stable controllers using all four models. All the controllers were able to react to known and unknown disturbances. Comparing all the results, the IR model in MPC is able to get the water level back to setpoint in the shortest time (15 min for known and 1 h for unknown disturbances) without large fluctuations in the discharge. For the other models these actions take at least 20 min and 100 min, in some cases with a number of low frequent oscillations in the discharge (IDF, IDZ). The poor performance of IDF is most probably due to the inaccurate description of the filter modeled as a pure delay.

Since the IR model contains the first oscillation mode, it is possible to achieve a higher performance. For the ID or IDZ models, using the same penalties as the ones applied in the IR

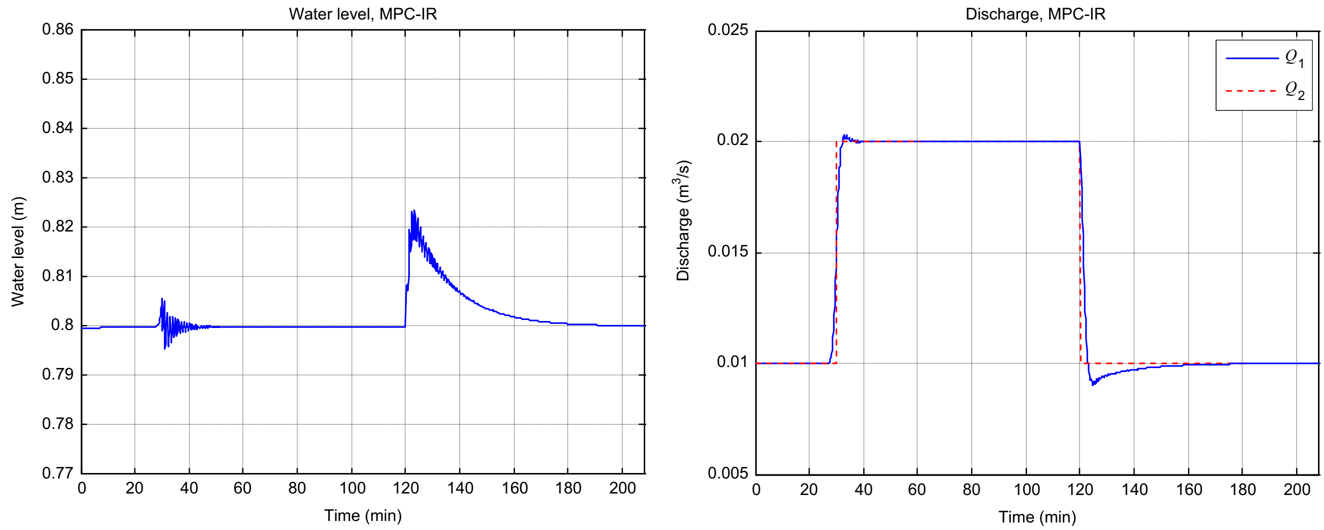


**Fig. 6.** Simulated distant downstream water level response ($h_2$) and control flow ($Q_1$) reacting on known and unknown disturbance of the model predictive controller using the integrator resonance model.

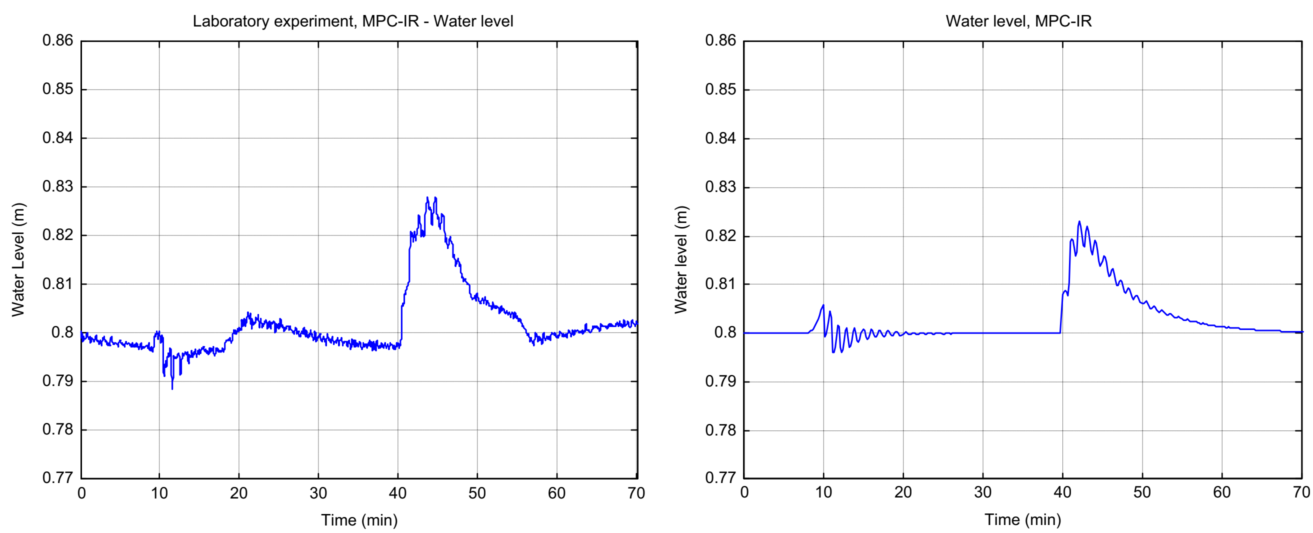


**Fig. 7.** Distant downstream water level response ($h_2$) of known and unknown disturbance of the model predictive controller using the integrator resonance model in a laboratory experiment (left) compared to simulation results (right).

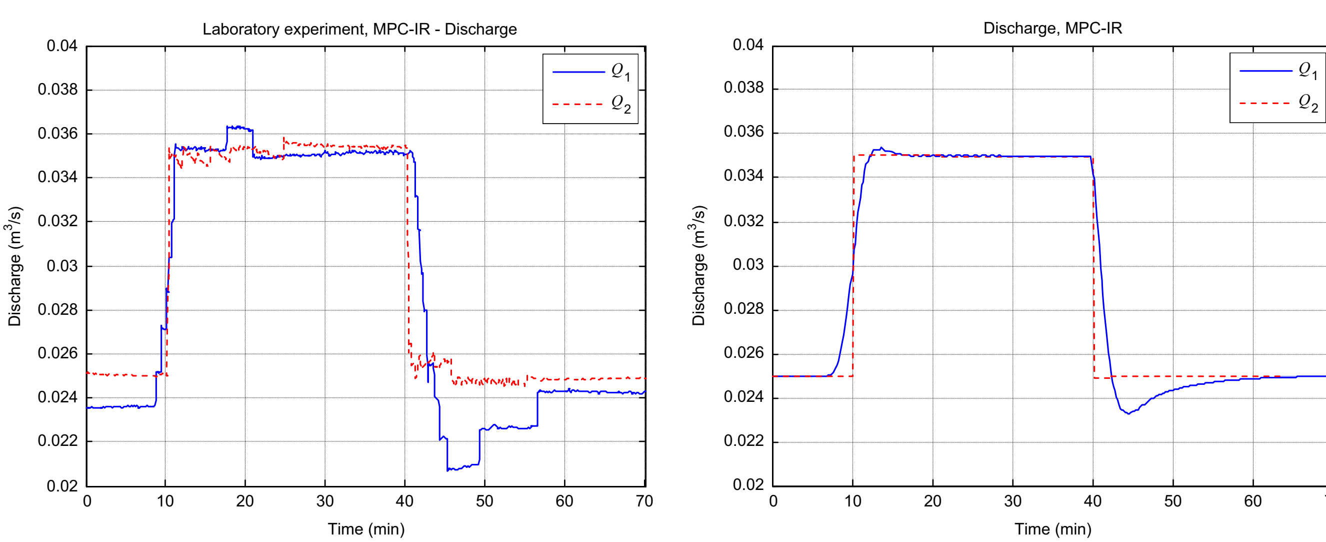


**Fig. 8.** Control flow ($Q_1$) reacting on known and unknown disturbance ($Q_2$) of the model predictive controller using the integrator resonance model in a laboratory experiment (left) compared to simulation results (right).

**Table 2**
Objective function value for all combinations of flows for which the IR model is designed and on which flow it is tested in closed loop.

| Flow (l/s) | Design flow (l/s) | | | |
|---|---|---|---|---|
| | 10 | 50 | 90 | 130 |
| 10 | 0.0640 | 0.0653 | 0.0651 | 0.0675 |
| 50 | 0.0640 | 0.0649 | 0.0646 | 0.0669 |
| 90 | 0.0657 | 0.0658 | 0.0645 | 0.0662 |
| 130 | 0.0684 | 0.0679 | 0.0664 | 0.0669 |

model, resulted in severe oscillations in the control flow indicating an unstable close loop system.

A final check on the general applicability of the IR model is a sensitivity analysis of the robustness. A test is set up in which the IR model is designed using the magnitude and frequency of the resonance peak belonging to the flows of 10, 50, 90 and 130 l/s. These models are tested on the numerical model running at different flows (again at 10, 50, 90 and 130 l/s). So for each flow test there is one IR model that corresponds to that flow and three that have a flow mismatch. Table 2 gives the results of the objective function value $J_{closedloop}$ over the entire simulation period $m$ for the 'known step' test (see Fig. 6, first 60 min) according to Eq. (9). The same objective function is used as in the previous tests (see Eq. (6)). Also the same penalties ($W_e$=100, $W_{esum}$=0.04, $W_{\Delta Q}$=10,000) are used

$$J_{closedloop} = \sum_{k=1}^{m} \{W_e \cdot e(k)^2 + W_{e_{sum}} \cdot e_{sum}(k)^2 + W_{\Delta Q} \cdot \Delta Q_1(k-1)^2\} \tag{9}$$

The result of this sensitivity test shows that the IR model is robust against flow changes. In general, the case where the flow for which the IR model is designed coincides with the flow on which it is tested shows the lowest objective function value, indicating the highest performance.

## 9. Conclusions

Model predictive control based on the integrator resonance model is tested on an accurate model of an open water channel and on the actual open water channel. Using the integrator resonance model is preferable for describing the relevant dynamics for controller design of open water channels that are short, flat and deep. This can be seen by comparing the IR model to the other simplified models known in the literature (integrator delay, integrator delay zero and integrator delay with filter) in a closed loop test using model predictive control on such an open water channel. The test with the MPC-IR controller executed on the actual laboratory canal pool compared to the same controller applied to the non-linear high-order Saint-Venant simulation model shows very similar results. This indicates that the modeling software is accurate enough for designing and testing controllers' offline.

## References


Camacho, E. F., & Bordons, C. (2004). *Model predictive control*. New York: Springer.

Chow, V. T. (1959). *Open-channel hydraulics*. New York: McGraw-Hill Book Co. Inc.

Figueiredo, J., Botto, M. A., & Rijo, M. (2013). SCADA system with predictive controller applied to irrigation canals. *Control Engineering Practice*, *21*(6), 870–886.

Li, Y., & De Schutter, B. (2011). Stability and performance analysis of an irrigation channel with distributed control. *Control Engineering Practice*, *19*(10), 1147–1156.

Litrico, X., & Fromion, V. (2009). *Modeling and control of hydrosystems*. Springer, ISBN: 9781848826236, London.

Litrico, X., & Fromion, V. (2004). Analytical approximation of open-channel flow for controller design. *Journal of Applied Mathematical Modelling*, *28*(7), 677–695.

Malaterre, P. O., & Baume, J. P. (1998). SIC 3.0, a simulation model for canal automation design. In *Proceedings from international workshop on regulation of irrigation canals: State of art of research and applications*. Marrakech, Morocco: Mokhlisse, A.

van Overloop, P.J. (2006). *Model predictive control on open water systems* (Doctoral dissertation). The Netherlands: Delft University of Technology.

van Overloop, P. J., Miltenburg, I. J., Bombois, X., Clemmens, A. J., Strand, R. J., Giesen, N. C. van de, & Hut, R. W. (2010). Identification of resonance waves in open water channels. *Control Engineering Practice*, *18*(8), 863–872.

Pascual, J., Romera, J., Puig, V., Cembrano, G., Creus, R., & Minoves, M. (2013). Operational predictive optimal control of Barcelona water transport network. *Control Engineering Practice*, *21*(8), 1020–1034.

Schuurmans, J. (1997) *Control of water levels in open channels* (Doctoral dissertation). The Netherlands: Delft University of Technology.

Sepúlveda Toepfer, C. (2008). *Instrumentation, model identification and control of an experimental irrigation canal* (Doctoral dissertation). Spain: Universitat Politècnica de Catalunya.

Vandevegte, J. (1990). *Feedback control systems*. New Jersey: Prentice-Hall.

Weyer, E. (2001). System identification of an open water channel. *Control Engineering Practice*, *9*, 1289–1299.