MULTIVARIABLE MODEL PREDICTIVE CONTROL OF WATER LEVELS ON A LABORATORY CANAL

Control predictive multivariable d'hauteurs d'eau dans un canal de laboratoire

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ABSTRACT

Automatic control of irrigation canals can reduce the loss of water in considerable amounts, therefore it is generating ecologic and economic benefits. There have been many different types of automatic controllers developed, but only few of them had the opportunity of being tested on the field due to the long delay time and the inconveniences of interrupting the operation of the irrigation. Therefore, the automatic controllers developed for large irrigation canals should be tested before by means of numerical simulations and/or laboratory experiments.
The Technical University of Catalonia possesses a laboratory irrigation canal with the length of 220m, with 3 motorized gates, and 11 level sensors that are connected to a SCADA system. This facility makes it possible to test controllers of any type, since all the instrumentation and real time operation runs within a flexible working environment running in Matlab-Simulink . The canal can be configured from one pool to three pools, which allows the development of multivariable control.
A numerical model of the canal has been developed using the 1D hydrodynamic model SIC. With the help of this software it is possible to simulate the hydraulics of the canal and, due to the link between SIC and Matlab, also to test any controller developed previously in the Matlab environment.
In this work a centralized multivariable model predictive controller for water levels is developed and validated by means of numerical simulation.

1. INTRODUCTION

Irrigation is one of the largest water consumers while its efficiency is generally very low. In case of traditional operation up to 40% of the irrigation water can be lost. The reduction of this water loss is not only beneficial for economic reasons, but also for ecological needs. Part of the losses is caused by the inappropriate management which can be reduced by introducing automation. The goal of automatic canal operation is to deliver the right amount of irrigation water at the right time, allowing on-demand operation of irrigation canals. This does not only make it more convenient for the users, but also allows them to use the least amount of water, just in the time and amount as they need it. Also, while discharges and water levels

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are regulated, construction and maintenance costs can be saved due to the lower fluctuations of the water levels.

2. THE LABORATORY OPEN CHANNEL

The laboratory canal (UPC-PAC: Technical University of Catalonia - Control Algorithms Test Canal) is located in Barcelona, at the Northern Campus of the University. The facility occupies 22.5m x 5.4m surface area: being 220m long and having serpentine shape. It is 1 m deep, 0.44 m wide, and contains 3 motorized vertical sluice gates, 9 water levels sensors, 4 rectangular weirs. With the help of the gates the canal is possible to be configured as a SISO (Single Input/Single Output) or MIMO (Multiple Input/Multiple Output) system. It has been built with zero slope in order to achieve the largest possible time delay, that in case of the normal operation discharge (70 l/s) is about 80 s. At the upstream end there is a constant level reservoir and at the downstream end there is a sharp crested weir with variable height.

Figure 1: The UPC-PAC: the laboratory canal of the Technical University of Catalonia

The water level measurement and the gate opening data is sent to the supervisory control and data acquisition (SCADA) system. It has been developed in Matlab-Simulink environment, therefore it is possible to test any control algorithms written in Matlab. The discharges are calculated from the measured variables using the hydraulic relationships at the gates and the weirs.

In this work the following configuration was used: (Figure 2) the canal was set as three pools, at the upstream end of Reach1 there is Gate 1 and at the downstream end Level 1 is controlled (that is just upstream of Gate 2), Reach 2 is limited by Gate 2 at the upstream end and Gate 3 at the downstream end, and the last Reach 3 has a constant height weir at the downstream end (W3). There are two offtakes in use (W1 and W2), both of them are gravity type, where the discharge varies with the water level.

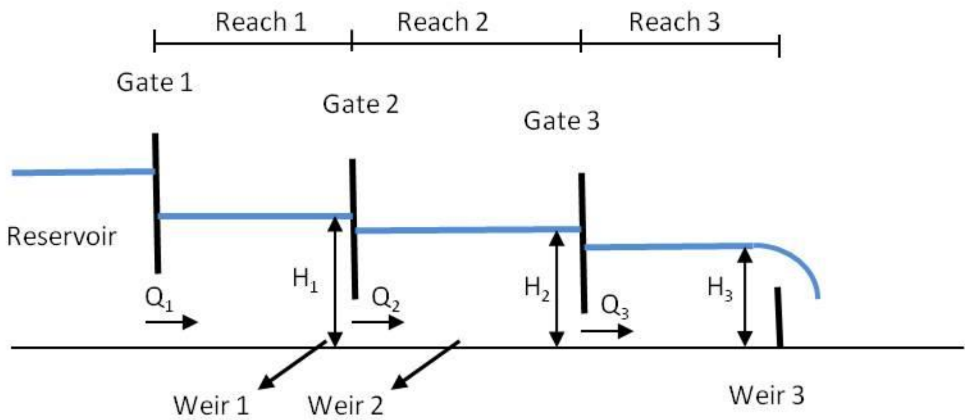

Figure 2: The UPC-PAC: the laboratory canal of the Technical University of Catalonia

Several control schemes had been developed and tested on the UPC-PAC before, like decentralized and centralized predictive control of water levels [1], model predictive control for discharge [2].

3. MODEL FOR CONTROL OF THE CANAL UPC - PAC

3.1 Linear model of a canal pool

Each reach of the canal is modelled using the integrator delay (ID) model. This simplified model was developed by Schuurmans [3] and it is widely used for control purposes [4, 5, 6]. It is based on differentiating the approximation of the uniform flow and that of the backwater part. Hence the two parts: the uniform flow part and a downstream backwater flow part.

Due to the backwater part, the dynamics are complicated. Reflecting waves are travelling up and down the reach. However, in low frequencies the water level integrates the discharge. Therefore the water level can be approximated as the integral of the flow, and the gain is approximated by the reciprocal of the backwater surface (see Equation 1).

In the uniform part the flow rate downstream is assumed equal to the upstream flow rate some time before.

In this case, since the bed slope is zero, all the canal reaches are affected by backwater. Therefore, the model assumes that the canal reach behaves as a tank, the water level is the integral of the discharge, and the time it takes for the upstream discharge to arrive to the downstream water level is the time delay. This behaviour can be described with the following equation in the Laplace domain:

$$h_i(s) = \frac{1}{A_e s} e^{-\tau s} q_i \quad s \tag{1}$$

where $h_i(s)$ is the downstream water level relative to a steady state water level in the i[th] reach, A_e is the backwater area, s is the Laplace's operator, τ is the time delay and $q_i(s)$ is the upstream discharge relative to a steady state discharge. The relative discharge and water level are defined as the following:

$$h_i = H_i - H_{0i} \tag{2}$$

$$q_i = Q_i - Q_{0i} \tag{3}$$

,

where H_i and Q_i are the measured values of the discharge and the water level respectively, and Q_0 and H_0 are the steady state values. From now on the steady state values of the variables are noted with underscored zero. The whole pool is supposed to be affected by backwater in this case, therefore the surface area of each canal pool is used as backwater area. Since the water profile is very close to horizontal in the case of the laboratory canal, the surface area is calculated as the length of the reach multiplied by the width. The time delay is estimated as:

$$\tau = \frac{L}{V_0 + C_0} \tag{4}$$

where V_0 is the velocity (belonging to the reference discharge) and C_0 is the celerity (calculated from the reference water depth) using the formula:

$$C_0 = \sqrt{gH_0} \tag{5}$$

$$A_e = H_0 L \tag{5}$$

where h is the reference water level relative to the bed level and g is the gravitational acceleration. Values for the three canal pools are given in Table 1. All these values were calculated based on the chosen reference discharge and reference water level (H_0). The reference discharge in all the cases was 70 l/s.

Reach	Length, L (m)	Water level H_0, (m)	Backwater area, A_e (m^2)	Time delay, τ (s)
1	87	0.8	32.28	31
2	90.2	0.6	39.68	32
3	43.3	0.45	19.93	16

Table 1: Parameters of the ID model of the three canal reaches of the UPC-PAC

After applying the z-transform the model can be described with the following linear time invariant discrete form:

$$h_i(k+1) = h_i(k) + A_d q_i(k-d+1) - A_d q_{i+1}(k) \tag{6}$$

where $h(k)$ is the downstream water level of the i^th^ reach at instant k and $h(k+1)$ is the downstream water level at instant $k+1$ and d is the delay steps, q_i is the inflow and q_{i+1} is the outflow of the i^th^ reach. A_d is calculated as:

$$A_d = \frac{T_d}{A_e} \tag{7}$$

where T_d is the sampling time and A_e is the backwater surface.

3.2 Filter design

Due to the very small bed slope (actually horizontal), the test canal is sensitive to reflecting waves (resonances). In order not to excite these waves in the canal, appropriate filtering is required in the control system. Schuurmans [3] proposed to filter the resonance wave with a first order low-pass filter. However, this introduces additional delay. In each canal pool this is about 100s, hence it caused a significantly slower control loop. Therefore, in this work for a sampling time of 10s, 11 delay steps are used in the first two and 10 in the last pool. The filter is applied to the water levels signals. The filter coefficient is calculated using the backwater surface (A_e), the frequency (ω_p) and the magnitude (M_p) of the first resonance peak according to [3]:

$$T_f = \sqrt{\frac{A_e M_p}{\omega_p}} \tag{8}$$

The magnitude and the frequency of the first resonance peak are obtained from the Bode plots of the three pools. These plots are obtained using the geometric characteristics of the canal using the frequency model of Litrico and Fromion [7]. The data and the calculated filter coefficients are summarized in Table 2.

The final discrete equation of the filter is the following:

$$h_{fil}(k) = f_c h_{fil}(k-1) + (1-f_c) h_{mes}(k) \tag{9}$$

where $h_{fil}(k)$ is the filtered water level at instant k, $h_{fil}(k-1)$ is the filtered water level at instant $k-1$ (one sampling time step before the present), f_c is the discrete filter coefficient and $h_{mes}(k)$ is the water level measured at the instant k (present time). The discrete filter coefficient is calculated from the continuous filter coefficient T_f using the following equation:

$$f_c = e^{-T_d/T_f} \tag{10}$$

Reach	Resonance frequency, ω_p (rad/s)	Magnitude of the resonance peak, M_p (s/m^2)	Filter coefficient, $_c$ (-)
1	0.1071	32.28	0.7795
2	0.0866	39.68	0.7549
3	0.1516	19.93	0.6308

Table 2: The resonance frequency, the magnitude of the first peak and the continuous filter coefficients for the three reaches of the UPC-PAC

The predictive controller is taking into account the combination of the ID model and the first order low pass filter which will be referred in the following as IDF (Integrator Delay Filter) model.

4. CONTROLLER DESIGN

4.1 Control strategy

Control strategies can be divided into two main groups: centralized and decentralized. The decentralized control strategy is normally based on simpler feedback control methods. There are separate controllers designed for each pool, not taking into consideration the interaction between pools. In some cases a higher level controller supervises the independent controllers. With this strategy, only suboptimal control is achievable. The disadvantage of this control strategy is that since the interactions between canal pools are not taken into account, the performance can degrade considerably. The perturbation caused by one controller can spread throughout the whole canal system, hence disturbance amplification can occur.

On the other hand, the centralized control strategy takes into account all the objectives that have to be fulfilled and a controller is designed using global information of the canal state. Compared to decentralized control, flows can be adjusted simultaneously at several gates resulting in a mass transfer of water in less time. The best control performance can be obtained with this strategy, however, it is computationally more complicated and more expensive to implement in practice.

There are several examples in the literature of centralized control, of which some of them are implemented in the field. They can be categorized by the different types of controllers, for example downstream feedback with feedforward control [8], LQR [9], LQG [10] or model predictive control [4, 11]. In this work centralized model predictive control is used.

4.2 Model Predictive Control

A centralized unconstrained model predictive controller has been developed. Equation (6) is used as internal model for each reach. The control variables are the discharge under the gate and then a gate inverse formula is used to calculate each three gate openings as function of submerged flow and the difference between upstream and downstream water level. The controlled variables are the three water levels downstream in the reaches. The states of the system are the water level error, the discharge in the previous control steps and the integral of the water level error. In order to design a multivariable MPC controller, a state-space description of the system is needed, so a linearized, discrete-time, state-space model of the canal is assumed in the general form:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_d V(k) \\ y(k) &= Cx(k) \end{aligned} \quad (11)$$

where $x(k)$ is the state vector, $u(k)$ is the input control vector, $y(k)$ is the vector of measured outputs which are to be controlled, $V(k)$ is the disturbance vector, and A, B, B_d, C are matrices of appropriate dimensions. (see Appendix) The index k counts time steps. By using a state-space model, the current information required for predicting ahead is represented by the state variable at the current time. The model is built using the following equations:

$$e_i(k+1) = e_i(k) + A_d q_i(k-d+1) - A_d q_{i+1}(k) \quad (12)$$

$$\Delta q_i(k+1) = q_i(k+1) - q_i(k) \tag{13}$$

$$e_{\text{int}\,i}(k+1) = e_{\text{int}\,i}(k) + e_i(k) \tag{14}$$

where q_i is the discharge and e_i is the error of the downstream water level in the i^{th} reach. Equation 12 is obtained by subtracting the setpoint from both sides of equation 6 and equation 13 defines the incremental discharge variable Δq_i. Equation 14 describes an integral variable of the error ($e_{int\,i}$) that integrates the error in order arrive zero steady state error.

In case of the last reach, the output discharge was calculated by using the linearized equation of the weir at the downstream end of the canal pool, with a gain of k_h (see the Appendix).

Due to the extra time delay introduced by the filter, the delay steps are 11, 11 and 10 respectively in the three reaches. Therefore the overall number of the states (the length of the vector $x(k)$) is 38. The dimension of $u(k)$ is 3. The predictive controller output is obtained by minimizing the error and the integral error on the prediction horizon. The input variables are the measured water levels at the downstream end of each pool.

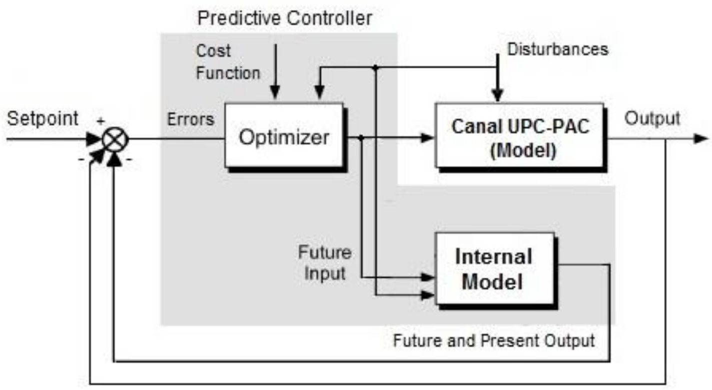

Figure 3: MPC system applied to the test canal

The prediction horizon was set to be long enough to exceed all the delays of the system, 35 steps with a sampling time of 10s. To obtain de control law, in order to keep the process as close as possible to a predefined reference, optimization process was carried out over the prediction horizon minimizing the following cost function:

$$\min_{\Delta u} J = \sum_{j=0}^{\lambda} x^T(k+j|k)\,Q x(k+j|k) + \sum_{j=0}^{\lambda} \Delta q^T(k+j|k)\,R \Delta q(k+j|k) \tag{15}$$

where e is a vector containing the water level errors for the three pools for the whole prediction horizon, superscript T means transpose of the matrix, Q is the weighing matrix of the state, Δq is a vector containing the inputs (change in discharge) for the whole prediction horizon and R is the weighing matrix for the input.

To minimize the function J, an optimizer predicts (calculates) output values as a function of past values of inputs and outputs and future control signals, making use of the internal model and substitutes these in the cost function, obtaining an expression whose minimization leads to the looked-for values. The first control action $\Delta q(k|k)$ is sent to the gates while the rest are neglected. This is because at the next sampling time the output $y(k+1)$ is measured by the system and then the optimization process is repeated with new values from an updated control sequence. Details about the formulation of the controller can be found in [5] and [12].

The calculated control variable of the MPC is the change in the input discharges. From the three discharge changes, the new gate openings are calculated by using the inverse of the gate equation. In the simulation tests, these gate openings are sent to a 1D hydrodynamic model, included in the SIC (Simulation of Irrigation Canals) software package [13].

4.2 Controller Tuning

The tuning parameters are the weights in the matrices and the length of the prediction horizon. In the weighing matrices, the weights were normalized. The same weights were given to all the pools for the error and the same penalization for the integral of the error. For the input, the weight on the first input was increased proportional to the gain of the first gate, since the gain of the first gate is much higher due to the high water level in the upstream reservoir. The weighing matrices are given below.

Weighing matrix for the input:

$$R = \begin{bmatrix} 8163 & 0 & 0 \\ 0 & 5102 & 0 \\ 0 & 0 & 5102 \end{bmatrix}. \quad (16)$$

Weighing matrix for the integral of the error:

$$Q_{\text{int}} = \begin{bmatrix} 0.166 & 0 & 0 \\ 0 & 0.166 & 0 \\ 0 & 0 & 0.166 \end{bmatrix}. \quad (17)$$

Weighing matrix for the error:

$$Q_e = \begin{bmatrix} 69.4 & 0 & 0 \\ 0 & 69.4 & 0 \\ 0 & 0 & 69.4 \end{bmatrix}. \quad (18)$$

5. RESULTS

5.1 Numerical tests

All tests were carried out by means of using the 1D hydrodynamic model: Simulation of Irrigation Canals (SIC) [13]. There are two different scenarios tested: setpoint tracking and disturbance rejection. In case of setpoint tracking, the setpoint was changed at 7 min. and after 25 min. it was changed again to the original one. This test was carried out in all the three reaches with all the three setpoints.

The disturbance rejection was tested by using the lateral weirs (Figures 5, 7 and 9). Initially two lateral offtakes were in “open” position (Weir 1 in the downstream end of Reach 1 and Weir 2 in the middle of Reach 2), the discharge over these weirs were 10 and 40 l/s respectively. In case of Reach 1, Weir 1 is located downstream of the reach. During the test for Reach 1 this offtake was “closed” from t=7 min. for 20 minutes, then opened again. In case of the test for Reach 2, the weir in the middle of the reach, Weir 2 was “closed” from t=7 minutes for 20 minutes, then opened again. In case of the last test the discharge in the downstream end of the last reach was increased at t=7 minutes by 40 l/s for 20 minutes.

In case of the setpoint changes (Figures 4, 6 and 8), the water levels returned to target level within 12 minutes. In case of the disturbances, the water levels recovered within 10 minutes. It can be seen how the controller starts the action before the change and arrives to the new state. The integrator action can be seen by the water level movements (oscillations), especially in the third reach. In all cases the steady state setpoint was reached.

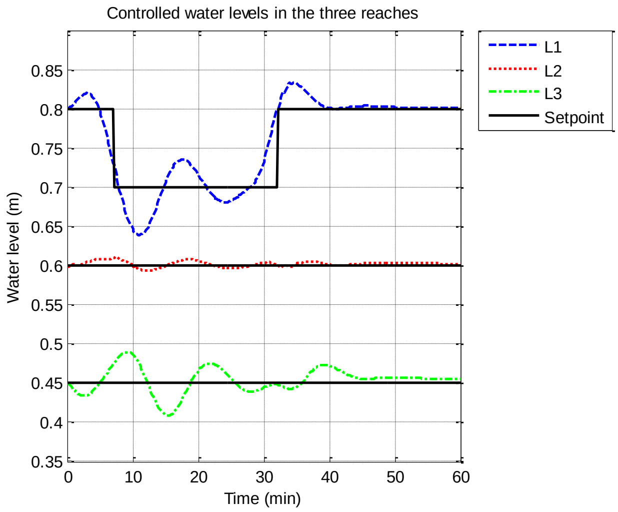

Figure 4. Reach 1:Level setpoint tracking

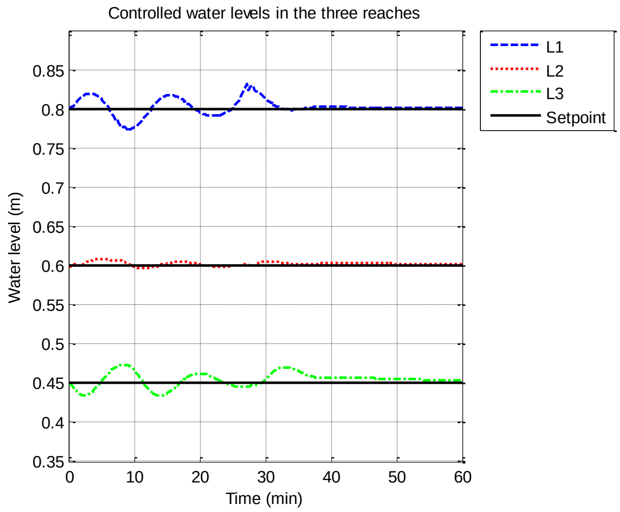

Figure 5. Reach 1: Discharge disturbance rejection

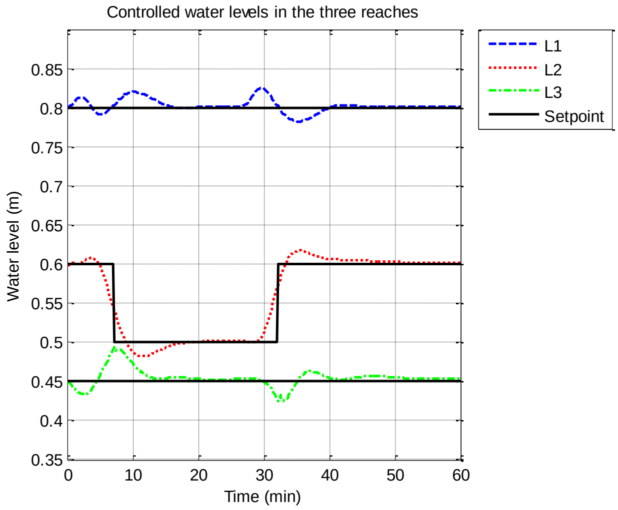

Figure 6. Reach 2: Level setpoint tracking

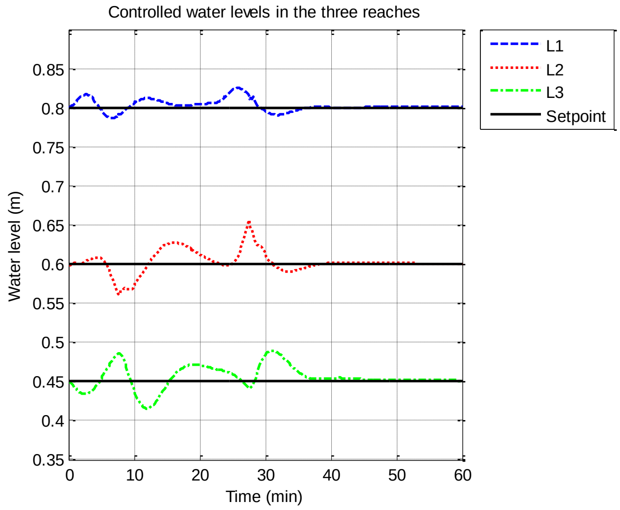

Figure 7. Reach 2: Flow disturbance rejection

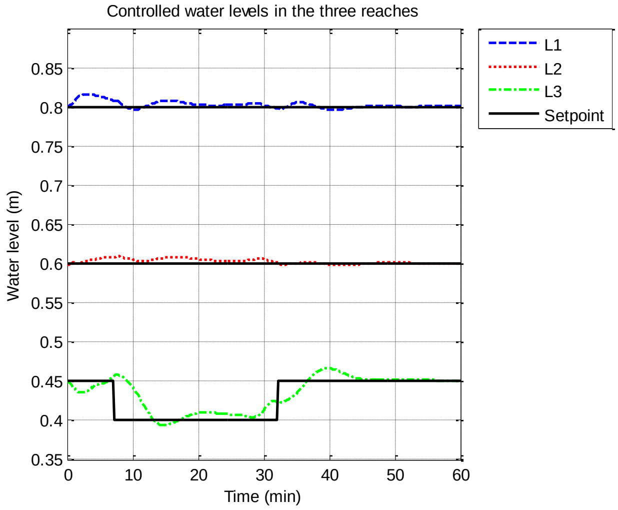

Figure 8. Reach 3: Level setpoint tracking

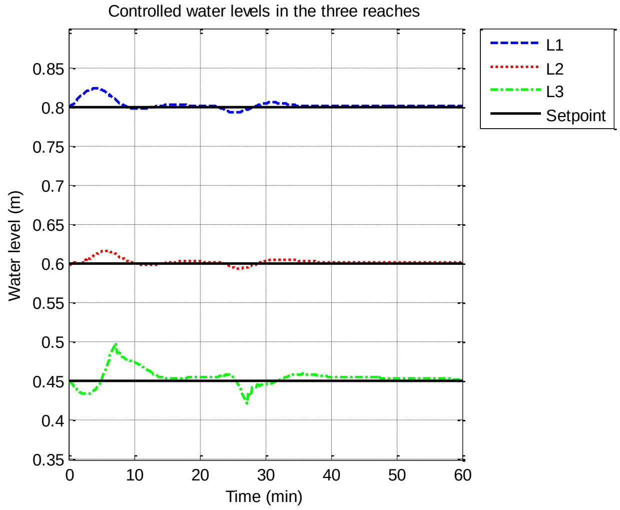

Figure 9. Reach 3: Flow disturbance rejection

5. CONCLUSION

Centralized model predictive controller are developed for a resonance sensitive laboratory canal. The controller has been tested numerically. For both presented scenarios, setpoint tracking and rejection of known disturbances as well, the controller showed an acceptable performance.

Future work will be implementation of the controller on the laboratory canal.

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APPENDIX

The matrices for the state space model

$$x(k) = \begin{bmatrix} q_1(k) & q_1(k-1) & q_1(k-2) & \ldots & q_1(k-10) & e_1(k) & q_2(k) & q_2(k-1) & q_2(k-2) & \ldots q_2(k-10) \\ e_2(k) & q_3(k) & q_3(k-1) & q_3(k-2) \ldots & q_3(k-10) & e_3(k) & e_{1\text{int}}(k) & e_{2\text{int}}(k) & e_{3\text{int}}(k) & \end{bmatrix}^T$$

$$u(k) = \begin{bmatrix} \Delta q_1(k+1) \\ \Delta q_2(k+1) \\ \Delta q_3(k+1) \end{bmatrix} \qquad \mathrm{V}(k) = \begin{bmatrix} q_{d1}(k+1) \\ \Delta h_{ref1}(k+1) \\ q_{d2}(k+1) \\ \Delta h_{ref2}(k+1) \\ q_{d3}(k+1) \\ \Delta h_{ref3}(k+1) \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -A_{d1} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -A_{d2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -A_{d1} & 0 \\ 0 & 0 & -A_{d2} \\ 0 & 0 & 0 \end{bmatrix} \qquad B_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -A_{d1} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -A_{d2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A_{d3} & 1 \\ -A_{d1} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -A_{d2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A_{d3} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix}
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0 & 0 & 0 & 0 & A_{d1} & 1 & -A_{d1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{d3} & 1-A_{d3}k_h & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A_{d1} & 1 & -A_{d1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{d2} & 1 & -A_{d2} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{d3} & 1-A_{d3}k_h & 0 & 0 & 1
\end{bmatrix}$$